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SOME NOTES ON THE THEORY OF PROBABILITY.

By Professor G. B. M. ZERR. A. M., Principal of High School, Staunton, Virginia.

Of the three memoirs written by Prevost and Lhuilier, the first entitled, "Sur les Probabilites," was read November 12th, 1795 and occupies pages 117-142 of the mathematical portion of the volume, "Memoires de l'Acad. .... Berlin. This important Memoir deals with the following venerable problem:

An urn contains m balls some of which are white and the rest black, but the number of each is unknown. Suppose p white balls and q black balls have been drawn and not replaced; required the probability that out of the next $r+s$ drawings r shall give white balls and s black balls.

This problem suggests the discussion of the more general problem: From an unknown number of balls each equally likely to be any of n colors $a_1+a_2+a_3+\dots\dots+a_n$ balls are drawn and turn out a_1 of the first color, a_2 of the second, a_3 of the third, $\dots\dots a_n$ of the nth. If $b_1+b_2+b_3+\dots\dots+b_n$ more balls are drawn find the probability that b_1 are of the first color, b_2 of the second, $\dots\dots$, b_n of the nth. The problem will not be altered, if we suppose the balls arranges along a straight line of length unity on n differant portions of the line. Call the first portion x_{n-1}, the sum of the first and second, x_{n-2}, the sum of the first three, $x_{n-3}, \dots$ the sum of the first $(n-1)$, x_1, then we get for the required chance the following definite integral:

$$p=\frac{\underline{b_1+b_2+b_3+\dots\dots+b_n}}{\underline{|b_1}\,\underline{|b_2}\,\underline{|b_3}\dots\dots\underline{|b_n}}\;\frac{\int_0^1\int_0^{x_1}\int_0^{x_2}\dots\dots\int_0^{x_{n-2}}(1-x_1)^{a_1+b_1}(x_1-x_2)^{a_2+b_2}(x_2-x_3)^{a_3+b_3}\dots\dots x_{n-1}^{a_n+b_n}dx_1dx_2dx_3\dots\dots dx_{n-1}}{\int_0^1\int_0^{x_1}\int_0^{x_2}\dots\dots\int_0^{x_{n-2}}(1-x_1)^{a_1}(x_1-x_2)^{a_2}(x_2-x_3)^{a_3}\dots\dots x_{n-1}^{a_n}dx_1dx_2dx_3\dots\dots dx_{n-1}}$$

$$=\frac{\underline{|b_1+b_2+b_3+\dots\dots+b_n}\;\underline{|a_1+b_1}\;\underline{|a_2+b_2}\;\underline{|a_3+b_3}}{\underline{|b_1}\;\underline{|b_2}\;\underline{|b_3}\dots\dots\underline{|b_n}\;\underline{|a_1}\;\underline{|a_2}\;\underline{|a_3}\dots\dots\underline{|a_n}}\;\frac{\dots\dots\underline{|a_n+b_n}\;\underline{|a_1+a_2+a_3+\dots\dots+a_n+n-1}}{\underline{|a_1+a_2+a_3+\dots\dots+a_n+b_1+b_2+b_3+\dots\dots+b_n+n-1}}.$$

When $n=2$,

$$p=\frac{\underline{|b_1+b_2}\;\underline{|a_1+b_1}\;\underline{|a_2+b_2}\;\underline{|a_1+a_2+1}}{\underline{|b_1}\;\underline{|b_2}\;\underline{|a_1}\;\underline{|a_2}\;\underline{|a_1+a_2+b_1+b_2+1}},$$ same result as given in the

memoir above referred to.

When $n=3$,

$$p=\frac{|\underline{b_1+b_2+b_3}\ |\underline{a_1+b_1}\ |\underline{a_2+b_2}\ |\underline{a_3+b_3}\ |\underline{a_1+a_2+a_3+2}}{|\underline{b_1}\ |\underline{b_2}\ |\underline{b_3}\ |\underline{a_1}\ |\underline{a_2}\ |\underline{a_3}\ |\underline{a_1+a_2+a_3+b_1+b_2+b_3+2}}$$

Let $\dfrac{|\underline{a_1+a_2+a_3+2}}{|\underline{a_1+a_2+a_3+b_1+b_2+b_3+2}}=A$, then the chance that all are of the first color is $p_1=\dfrac{|\underline{a_1+b_1+b_2+b_3}}{|\underline{a_1}}A$. $p_2=\dfrac{|\underline{a_2+b_1+b_2+b_3}}{|\underline{a_2}}A$ that all are of the second color. $p_3=\dfrac{|\underline{a_3+b_1+b_2+b_3}}{|\underline{a_3}}A$. that all are of the third color.

$p_4=\dfrac{|\underline{b_1+b_2+b_3}\ |\underline{a_1+b_1+b_2}\ |\underline{a_3+b_3}}{|\underline{b_1+b_2}\ |\underline{b_3}\ |\underline{a_1}\ |\underline{a_3}}A$, that b_1+b_2 are of the first color and b_3 of the third.

$p_5=\dfrac{|\underline{b_1+b_2+b_3}\ |\underline{a_1+b_3}\ |\underline{a_3+b_1+b_2}}{|\underline{b_1+b_2}\ |\underline{b_3}\ |\underline{a_1}\ |\underline{a_3}}A$, that b_3 are of the first color and b_1+b_2 of the third and so on for any combination.

When $a_1=5$, $a_2=3$, $a_3=2$, $b_1=b_2=b_3=1$.

Then $p=\frac{72}{455}$, $p_1=\frac{8}{65}$, $p_2=\frac{4}{91}$, $p_3=\frac{2}{91}$, $p_4=\frac{9}{65}$, $p_5=\frac{36}{455}$. The chance that there are none of the second color is $p_6=p_1+p_3+p_4+p_5=\frac{33}{91}$.

These numerical results are the same as those obtained by an algebraic solution of the same problem given by the late Professor Wolstenholme.

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NON-EUCLIDEAN GEOMETRY: HISTORICAL AND EXPOSITORY.

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[Continued from the March Number.]

PROPOSITION XVI. *By any quadrilateral ABCD, of which the four angles together are equal to, or greater, or less than four right angles, is established respectively the hypothesis of right angle, or obtuse angle, or acute angle.*